An Improved Algorithm for Code Equivalence

Julian Nowakowski **Ruhr University Bochum, Germany**

Code Equivalence

Linear [n,k] code:

Monomial:

Code Equivalence

Linear [n,k] code: subspace $C \subseteq \mathbb{F}_q^n$ of dimension k.

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Examples:

- permutations $\mathbf{P}$
- diagonal matrices $\mathbf{D}$ with non-zero diagonal

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

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Fact:

Every monomial is of the form $\mathbf{Q} = \mathbf{P} \cdot \mathbf{D}$.

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$= \mathrm{wt}(\mathbf{v})$ for every $\mathbf{v} \in \mathbb{F}_q^n$.

Codes C_1, C_2 are linearly equivalent.

$$\Longleftrightarrow$$

$C_2 = C_1 \cdot \mathbf{Q}$, for some monomial $\mathbf{Q}$.

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- $n \in \{252,400,548\},\ k = n/2,\ q = 127$

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Quantum algorithm with runtime $2^{n/3}$.

Comparison with State of the Art

Short codeword based:

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- Beullens, 2020
- Barenghi, Biasse, Persichetti, Santini, 2023

Canonical form based:

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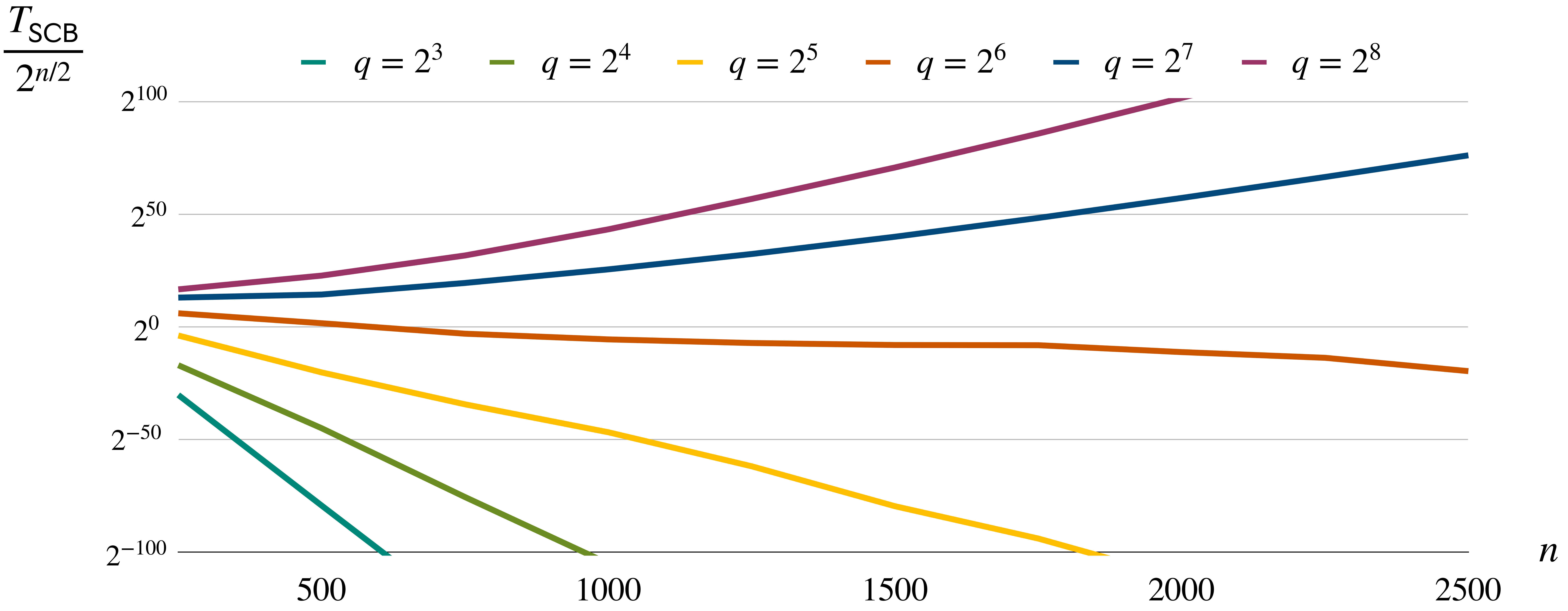

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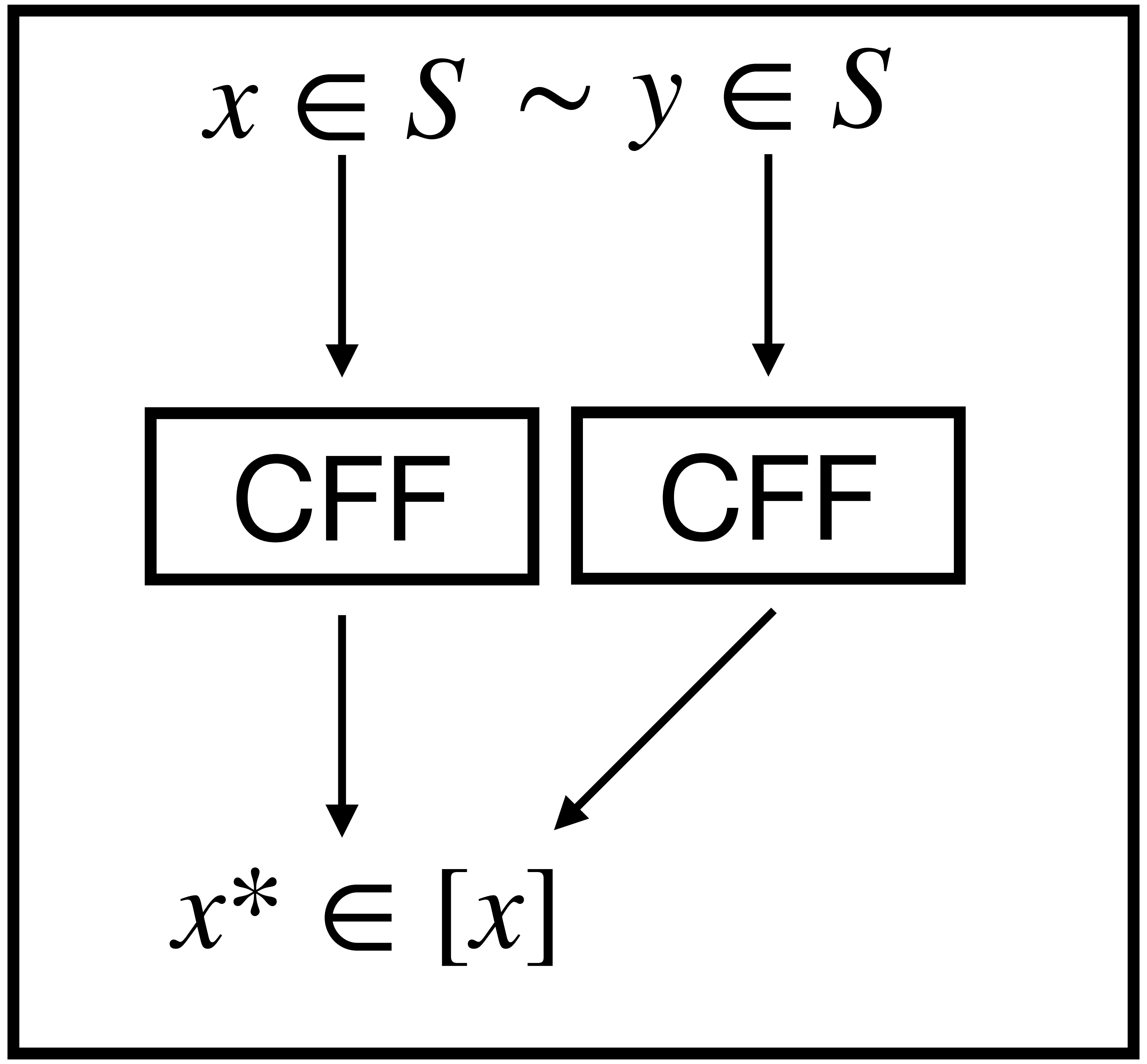

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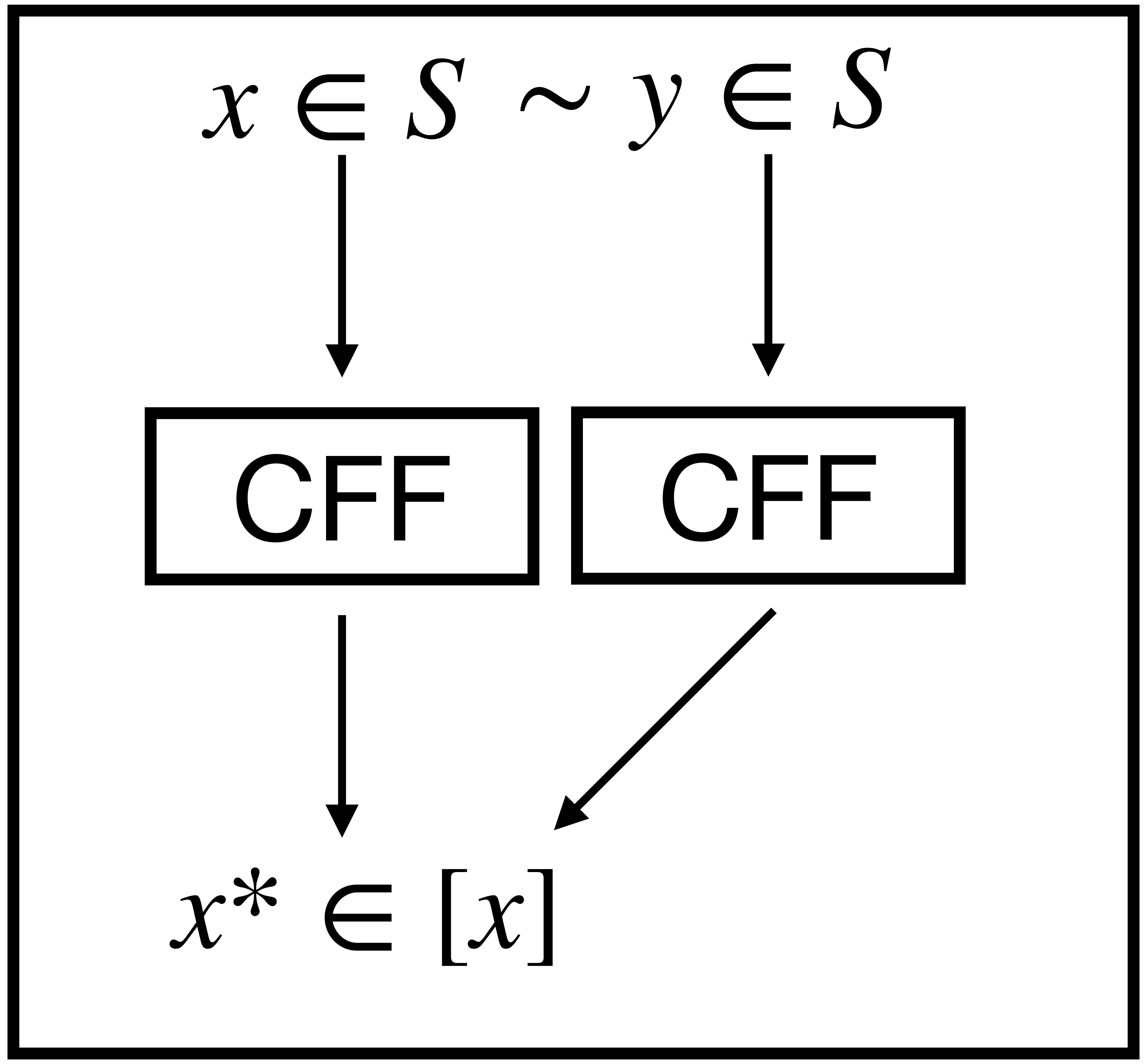
Example:

- $S = \mathbb{Z}$
- $x \sim y \iff x \equiv y \mod 3$
- CFF : $\mathbb{Z} \to \{0,1,2\}, \quad x \mapsto x \bmod 3$



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\begin{tikzcd}
x \in S \arrow[d] & \sim & y \in S \arrow[d] \\
\boxed{\text{CFF}} \arrow[d] & & \boxed{\text{CFF}} \arrow[dll] \\
x^* \in [x] & &
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- $S := \mathbb{F}_q^{k\times(n-k)}$
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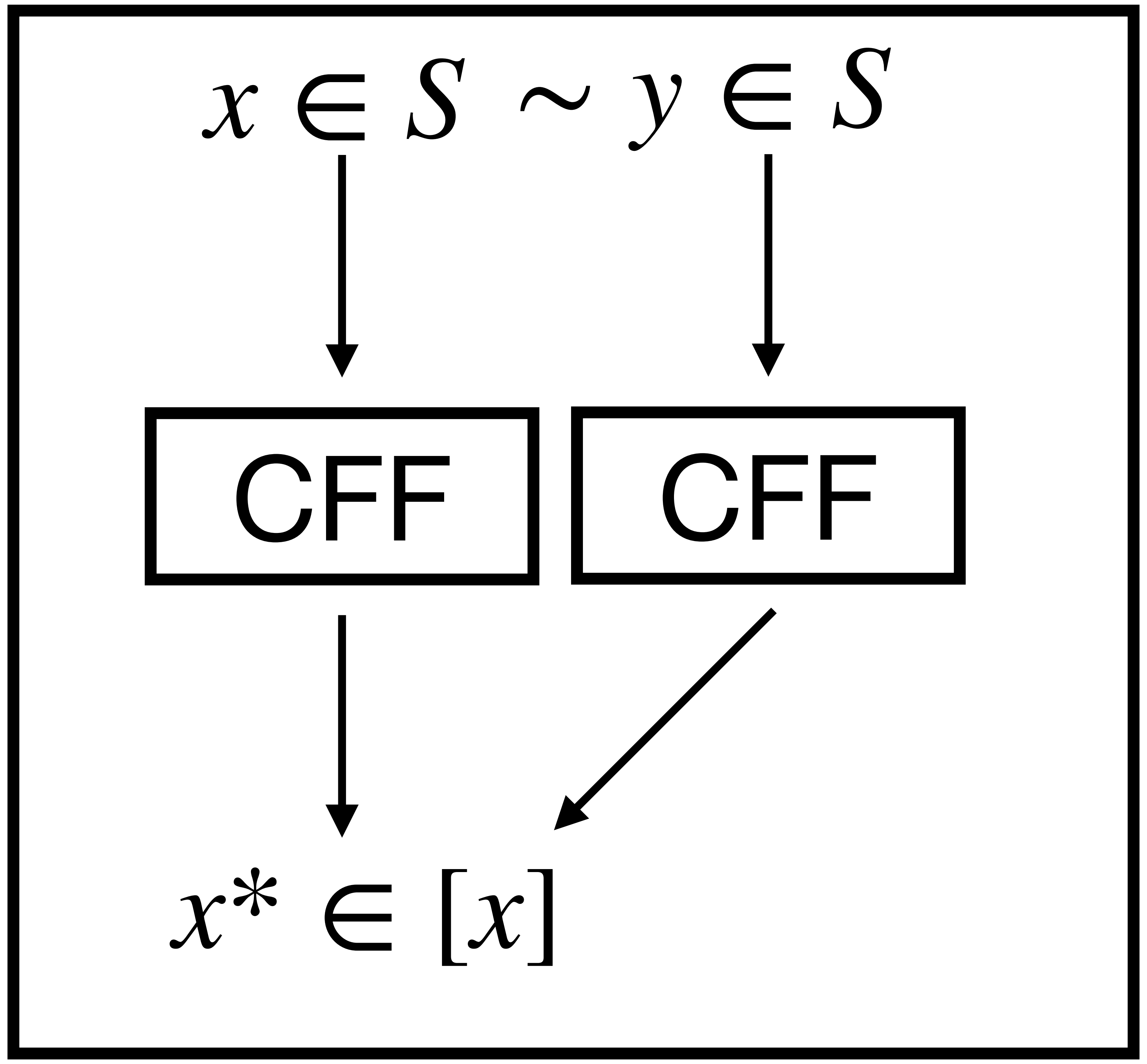

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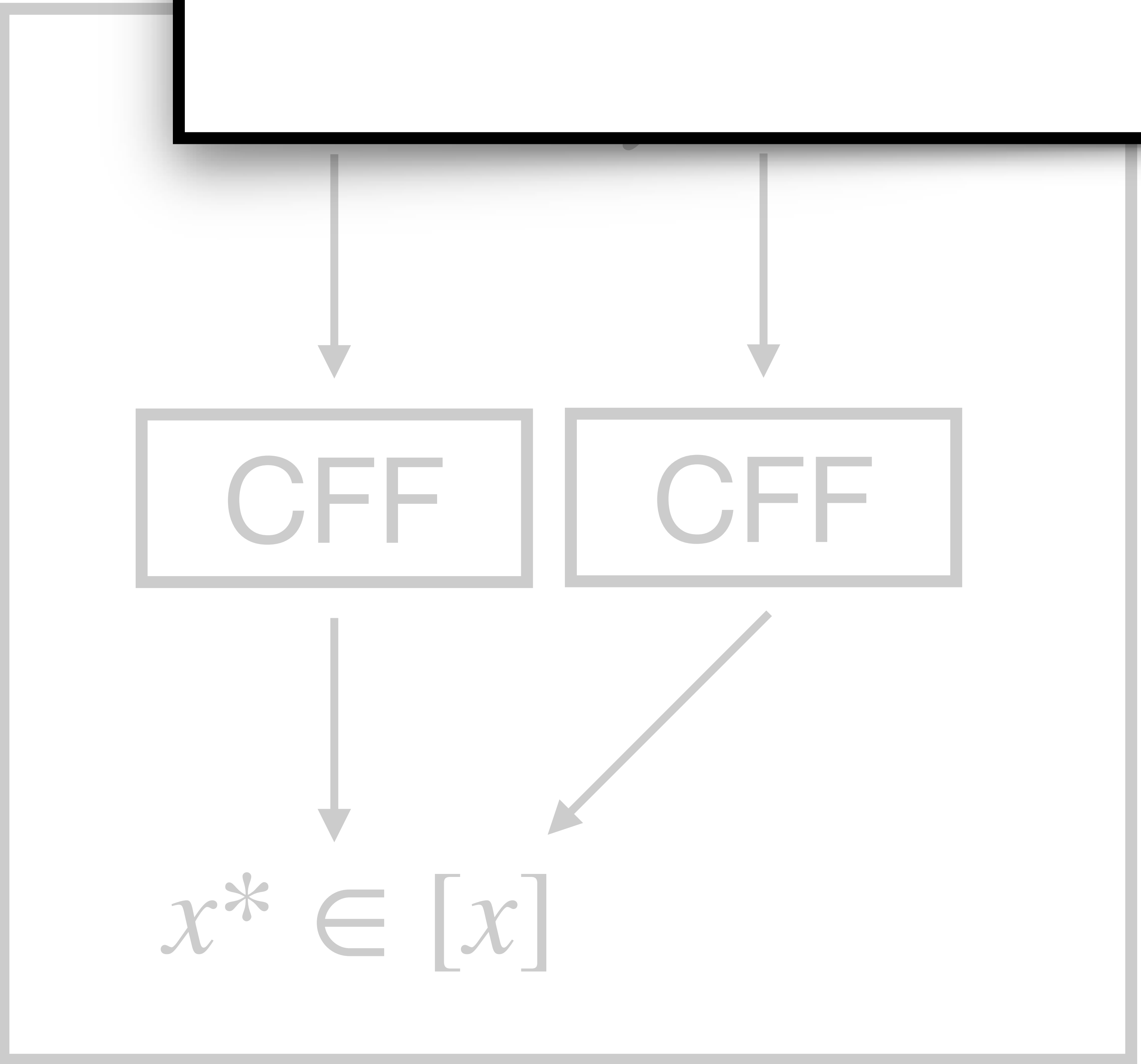
CPS 2025:

Suppose there exists an efficient CFF for LRL equivalence with success probability γ. Then we can solve the code equivalence problem in time

$$\gamma^{-1/2} \cdot 2^{n/2}.$$



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New result:
CFF with

$$\gamma = 1 - O(n^{-1})$$

for all $q \geq 7$.

succeed with probability $\gamma = \Theta(1)$.

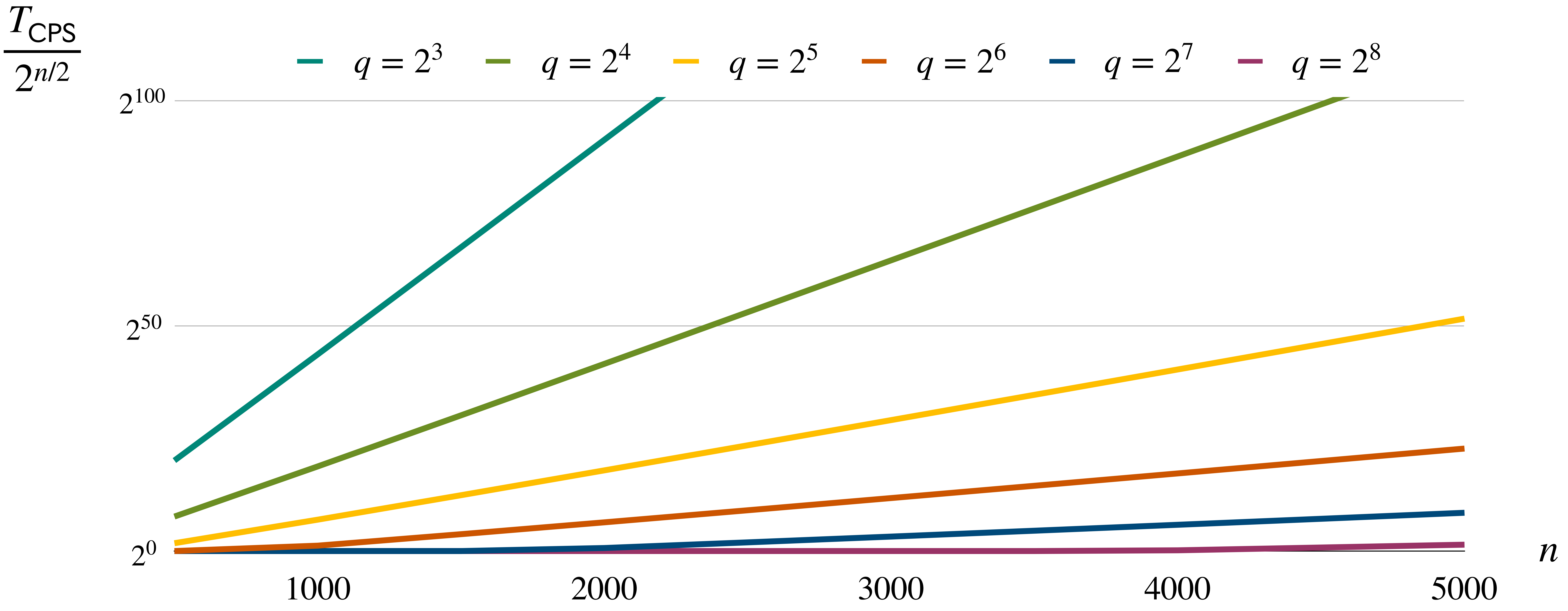
Comparison with CPS

Runtime using new CFF:

$$2^{n/2}$$

Runtime using CPS' CFF:

$$2^{n/2} \cdot 2^{\Theta(n)}$$



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Success Probability in Practice

q \ n	50	60	70	80	90	100
2	0 %	0 %	0 %	0 %	0 %	0 %
3	0 %	0 %	0 %	0 %	0 %	0 %
4	52 %	32 %	30 %	20 %	10 %	0 %
≥ 5	100 %	100 %	100 %	100 %	100 %	100 %

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  - Exponential improvement over short-codeword based for all $q \geq 2^7$
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https://ia.cr/2024/1272

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- **Potential application**
  - Using improved CFF to construct more efficient code-based cryptosystems with smaller q

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